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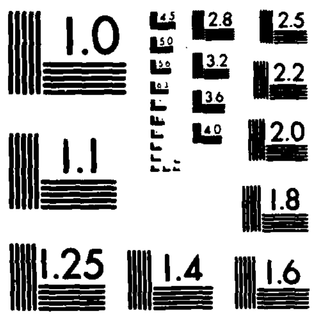

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NEW THEORY FOR THICK, COMPOSITE-MATERIAL RINGS

by

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NEW THEORY FOR THICK, COMPOSITE-MATERIAL RINGS*

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ABSTRACT

A new theory for the in-plane elastic behavior of shear-deformable ring-type structures, i.e., curved beams and complete rings, is presented. This theory falls between Bresse-Timoshenko-type ring theory, in which the shear deformation is assumed to be distributed uniformly through the thickness and corrected by a shear correction factor determined in an *ad hoc* fashion, and two- or three-dimensional elasticity theory. As an example, the theory is applied to the problem of a diametrically loaded thick ring. The predicted normal-stress distribution is in excellent agreement with published results in the literature obtained by photoelastic measurements.

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NOTATION

A - stretching stiffness

a - cross-sectional area of ring

B, B', C - bending-stretching coupling stiffnesses

D, F - bending stiffnesses

E - Young's modulus

G - shear modulus

h - total in-plane depth of ring

M - bending moment

N - circumferential force

P - concentrated force

p_i - internal pressure

Q - shear force

R - mean radius of ring

S - shear stiffness

V - circumferential displacement

v - circumferential displacement of middle surface

W - radial displacement

w - radial displacement of middle surface

y - circumferential position on middle surface

z - outward normal position coordinate

α, β - linear and quadratic coefficients in expansion of V in powers of z

γ - shear strain

γ^0 - transverse shear strain at $z = 0$

ε - normal strain

ε^0 - circumferential normal strain at $z = 0$

θ - circumferential position angle

κ - bending curvature change

ν - Poisson's ratio

σ - normal stress

τ - shear stress

ϕ - cubic coefficient in expansion of V in powers of z

$(\ )_{,y}$ - $\partial(\ )/\partial y$

1. INTRODUCTION

Ring-type structures are an important class of structural elements used as flywheel rims, shell rims, load-calibration rings, elements of transducers, and as circumferential reinforcements for cylindrical shells. Furthermore, the theory of rings can be considered to be a degenerate version of shell theory.

The theory of in-plane vibration of relatively thick circular rings, including transverse shear deformation and rotatory inertia, dates back to the work of Bresse (1859), which predates the classical thin-ring theory of Hoppe (1871) as well as Timoshenko's shear-deformable beam theory (1921). A discussion of some of the large body of ensuing literature was provided by Haines (1974). The pioneering shell theory due to Hildebrand et al. (1949) and the work of Benzley et al. (1973) should also be mentioned. An alternative is to use the two- or three-dimensional theory of elasticity, such as used by Federhofer (1935) and Endo (1972), respectively.

It has long been known that the Bresse-Timoshenko theory suffers from a serious flaw: it is based on the assumption of a uniform distribution of transverse shear strain through the thickness, whereas the actual distribution is that of a distorted parabola. Numerous *ad hoc* approaches, both static and dynamic, have been suggested for determining the so-called shear correction factor to account for the differences between these stress and strain distributions; see, for instance, Mindlin and Deresiewicz (1954) and Cowper (1966).

Only recently was an entirely new theory proposed to allow a more realistic shear strain distribution; it is due to Levinson (1981). This theory and its companion plate theory (Levinson, 1980), as well as analogous earlier theories by Ambartsumyan (1970), Reissner (1975), and Schmidt (1977), were derived for macroscopically homogeneous material only.

It is well known that transverse shear deformation plays an even more important role in structures composed of fiber-reinforced composite materials than in those of homogeneous material; see, for example, Tarnopol'skii et al. (1965). Dynamic and static derivations of the shear correction factor for a laminate were presented by Yang et al. (1966) and by Whitney (1973) and Bert (1973). The latter work was recently extended to the case of beams laminated of bimodular materials (different elastic properties in tension and compression) by Bert and Gordaninejad (1983).

Levinson's theory of plates was recently extended to laminated plates by Murthy (1981); see also Bert (1983). However, the Levinson theory cannot be extended to ring-type members and still satisfy the requirements of zero shear strain at the intrados and extrados of the ring. This is the motivation for the present theory, in which these zero-shear-strain requirements are satisfied.

Most of the theories of shear-deformable ring mechanics (including Bresse, 1859 and Haines, 1974) are what might be termed first-approximation theories, analogous to Love's first-approximation shell theory (see Kraus, 1967), in which it is assumed in the strain-displacement relations that $z/R \ll 1$. This is analogous to Winkler-Bach curved beam theory (see Seely and Smith, 1952) but in contrast to the shear-deformable ring theory due to Kirkhope (1977).

2. HYPOTHESES

In the present work, the following simplifying assumptions are made:

1. As in other ring theories, thickness normal strain is neglected, i.e., $\varepsilon_z = 0$.

2. The shear strain distribution is nonlinear in such a way that the

shear strain vanishes at the intrados and extrados of the ring. This is in contrast to all other ring or curved-beam theories (except elasticity theory) known to the present investigator, and allows accurate predictions without the use of a shear correction factor.

3. The theory is a first-approximation one, i.e., $z/R \ll 1$. This means that the theory should be most applicable to thin rings made of highly shear-deformable material (such as fiber-reinforced composite material).

4. Displacements are assumed to be sufficiently small that linear strain-displacement relations are adequate.

5. The ring cross section may be homogeneous or arbitrarily laminated of linear cylindrically orthotropic elastic material (see Fig. 1).

3. THEORETICAL DERIVATION

The following displacement field is used as the point of departure (Fig. 1):

$$
\begin{aligned}
V(\theta,z) &= v(\theta) + z\alpha(\theta) + z^2\beta(\theta) + z^3\gamma(\theta) \\
W(\theta,z) &= w(\theta)
\end{aligned}
\tag{1}
$$

The expression for circumferential displacement V contains the same terms used in Levinson's beam theory (1981) plus middle-surface displacement v to account for bending-stretching coupling and the quadratic term, which is now necessary. The expression for radial deflection W is consistent with hypothesis 1.

It is shown in the Appendix that it is not possible to use the displacement field (1) in conjunction with the exact linear strain-displacement relations and still satisfy hypothesis 2. Thus, consistent with hypotheses 3 and 4, the following first-approximation strain-displacement relations are used.

$$
\begin{aligned}
&\varepsilon_\theta = \varepsilon_y = (W/R) + V_{,y} \quad ; \quad \varepsilon_z = W_{,z} \\
&\gamma_{\theta z} = \gamma_{yz} = W_{,y} - (V/R) + V_{,z}
\end{aligned}
\tag{2}
$$

where $(\ )_{,y} = \partial(\ )/\partial y$, etc. Substituting the displacement field (1) into (2), one obtains the strain field

$$\varepsilon_y = [v_{,y} + (w/R)] + z\alpha_{,y} + z^2\beta_{,y} + z^3\delta_{,y} \quad ; \quad \varepsilon_z = 0$$
$$\gamma_{yz} = [w_{,y} - (v/R) + \alpha] + [2\beta - (\alpha/R)]z + [3\delta - (\beta/R)]z^2 - (\delta/R)z^3 \tag{3}$$

The material stress-strain relations for each layer in the laminate, in view of hypothesis 5, are

$$\sigma_y = E\varepsilon_y \quad ; \quad \tau_{yz} = G\gamma_{yz} \tag{4}$$

In view of the third of (3) and the second of (4), enforcement of vanishing shear strains at the extrados and intrados, i.e., $\tau_{yz}(y, \pm \frac{1}{2} h) = 0$, requires that

$$[w_{,y} - (v/R) + \alpha] + [3\delta - (\beta/R)](h/2)^2 = 0$$
$$[2\beta - (\alpha/R)] - (\delta/R)(h/2)^2 = 0 \tag{5}$$

Consistent with hypothesis 3, one can neglect $(1/8)(h/R)^2$ compared with unity. Within this approximation, it can be shown that equations (5) are satisfied if one sets

$$\beta = (\alpha/2R) - (1/6R)[w_{,y} - (v/R) + \alpha]$$
$$\delta = -(4/3h^2)[w_{,y} - (v/R) + \alpha] \tag{6}$$

Thus, the final expressions for the non-zero strain distributions can be written as

$$\varepsilon_y = [v_{,y} + (w/R)] + z\alpha_{,y} + (z^2/6R)[2\alpha_{,y} - w_{,yy} + (v_{,y}/R)]$$
$$- (4/3)(z^3/h^2)[w_{,yy} - (v_{,y}/R) + \alpha_{,y}] \tag{7}$$
$$\gamma_{yz} = [1 - (1/3)(z/R) - 4(z/h)^2 + (1/6)(z/R)^2 + (4/3)(z^3/Rh^2)]$$
$$\cdot[w_{,y} - (v/R) + \alpha] - (\alpha/2)(z/R)^2$$

The nonlinearity of the strain distributions in (7) is readily apparent.

As is customary, the stress resultants and stress couple are defined as (see Fig. 1)

$$(N,M) \equiv \int_a (1,z)\sigma_y \, da \quad ; \quad Q \equiv \int_a \tau_{yz} \, da \tag{8}$$

From the theory of elasticity in plane polar coordinates $(R+z,\theta)$, the circumferential and radial equations of equilibrium for any arbitrary point within the ring are, in the absence of body forces,

$$\begin{aligned} &\tau_{\theta z,z} + (\sigma_{\theta,\theta} + 2\tau_{\theta z})/(R+z) = 0 \\ &\tau_{\theta z,\theta} + (R+z)\sigma_{r,z} + \sigma_r - \sigma_\theta = 0 \end{aligned} \tag{9}$$

Multiplying the first of (9) by $(R+z)$, integrating with respect to z, using relations (6), and noting that $\tau_{\theta z}$ vanishes at the intrados and extrados, one has

$$N_{,y} + (Q/R) = 0 \tag{10}$$

Similarly, multiplying the first of (9) by $(R+z)z$ and integrating with respect to z, one obtains

$$M_{,y} - Q = 0 \tag{11}$$

Finally, integrating the second of (9) with respect to z, one finds

$$Q_{,y} - (N/R) + p_i = 0 \tag{12}$$

Substituting the strains (7) into the stress-strain relations (4) and thence into the generalized force definitions (8), one obtains the ring constitutive relations as follows

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B' \\ B & D \end{bmatrix} \begin{Bmatrix} v_{,y} + (w/R) \\ \alpha_{,y} \end{Bmatrix} - \begin{Bmatrix} C \\ F \end{Bmatrix} \left(w_{,y} - \frac{v}{R} + \alpha\right)_{,y} \tag{13}$$

$$Q = S[w_{,y} - (v/R) + \alpha] - S'\alpha \tag{14}$$

where the ring stiffnesses are defined as

$$(A,B) \equiv \int_a (1,z)E\,da \quad ; \quad (C,F) \equiv \int_a (1,z)[(z^2/6R) + (4/3h^2)z^3]E\,da$$

$$(B',D) \equiv \int_a (z,z^2)[1 + (z/2R)]E\,da \quad ; \quad S' \equiv (1/2)\int_a (z/R)^2 G\,da \tag{15}$$

$$S \equiv \int_a [1 - (z/3R) - 4(z/h)^2 + (1/6)(z/R)^2 + (4/3)(z^3/Rh^2)]G\,da$$

Equations (13) and (14) can be written in more compact form as follows:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B' \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^o \\ \kappa \end{Bmatrix} - \begin{Bmatrix} C \\ F \end{Bmatrix} \gamma^o_{,y} \tag{16}$$

$$Q = S\gamma^o - S'\alpha \tag{17}$$

Substituting the ring constitutive relations (13) and (14) into the equilibrium equations (10)-(12), one obtains the following displacement equations of equilibrium in matrix-differential operator form:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} v/R \\ \alpha \\ w/R \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ -p_i \end{Bmatrix} \tag{18}$$

Here,

$$L_{11} \equiv (AR + C)d_y{}^2 - (S/R) \quad , \quad L_{12} \equiv (B' - C)d_y{}^2 + (S/R) - (S'/R)$$

$$L_{13} \equiv - CRd_y{}^3 + (A + S)d_y \quad , \quad L_{21} \equiv (BR + F)d_y{}^2 + S$$

$$L_{22} \equiv (D - F)d_y{}^2 - S + S' \quad , \quad L_{23} \equiv - FRd_y{}^3 + (B - SR)d_y$$

$$L_{31} \equiv -[(A+S)+(C/R)]d_y \quad , \quad L_{32} \equiv [S - S' - (B'/R) + (C/R)]d_y \quad ,$$

$$L_{33} \equiv (SR + C)d_y^2 - (A/R) \quad , \quad d_y \equiv d(\quad)/dy \tag{19}$$

4. APPLICATION AND COMPARISON WITH EXPERIMENTAL RESULTS

As an application of the new theory, a diametrically loaded thick circular ring (see Fig. 2) is considered. Application of statics to the free-body diagram shown in Fig. 3 yields the following expressions for the generalized forces:

$$N = -P\cos\theta \quad , \quad Q = -P\sin\theta \quad , \quad M = -M_1 + PR\cos\theta \tag{20}$$

These expressions satisfy the equilibrium equations of the present theory, equations (10)-(12).

Since the problem is statically indeterminate to the first degree, an additional equation relation to deformation is required. From the ring constitutive relation, equation (13), one has

$$\alpha_{,y} = [AM - BN + (AF - BC)(Q_{,y}/S)]/[AD - BB' - (AF - BC)(S'/S)] \tag{21}$$

Integrating symbolically, one obtains

$$\alpha(\theta) = \int_0^\theta \alpha_{,y} R\, d\theta + \alpha(0) \tag{22}$$

Inserting the appropriate quantities into equation (22) and enforcing the symmetric conditions

$$\alpha(0) = \alpha(\pi/2) = 0 \tag{23}$$

one finds the following expression for the statically indeterminate bending moment:

$$M_1 = (2/\pi)[R + (B/A) - (1/ARS)(AF - BC)]P \tag{24}$$

It is noted that in the case of a homogeneous beam in thin ring theory ($S \to \infty$), equation (24) reduces to the classical value, $M_1 = (2/\pi)PR$.

The author was not able to locate any experimental stress or deflection data for a circumferentially-wound, fiber-reinforced composite-material ring, which would be the most stringent test for the new theory. However, some photoelastically obtained stress data were reported by Srinath and Acharya (1954) for a very thick ring of homogeneous, isotropic material. Specific data for their ring are listed in Table 1.

For the present special case of a homogeneous, isotropic ring, the ring stiffness parameters can be expressed as follows:

$$A/E = h \quad , \quad B/E = 0 \quad , \quad B'/E = h^3/24R \quad , \quad C/E = B'/3E \quad , \quad D/E = h^3/12 \quad ,$$
$$F/E = D/5E \quad , \quad S/E = (1/3)h/(1+\nu) \quad , \quad S'/E = (1/48)(h^3/R^2)/(1+\nu) \tag{25}$$

It was necessary to normalize the stiffnesses with respect to Young's modulus (E), since the actual value of the Young's modulus was not reported by Srinath and Acharya. The resulting expressions for the circumferential normal stress distribution are given below.

For the actual value of shear stiffness (S):

$$\sigma_y = -525 + 16{,}015\,z + 6{,}548\,z^2 + 13{,}195\,z^3 \tag{26}$$

Here, z and σ_y have units of inches and psi, respectively.

Neglecting transverse shear flexibility ($S \to \infty$):

$$\sigma_y = -517 + 16{,}104\,z + 6{,}315\,z^2 - 10{,}477\,z^3 \tag{27}$$

It is noted that inclusion of transverse shear deformation results in a more nonlinear stress distribution. A comparison of these predicted results with the reported experimental values is presented in Table 2. Certainly the results for the new theory (actual S) are encouraging. Also presented is the prediction of Winkler-Bach ring theory, for which the bending stress is given by

$$\sigma_b = (M/he)(e + z)/(R + z) \tag{28}$$

where e is the distance from the centroid of the cross section to the neutral surface and is given by

$$e = R - h\left[\ln \frac{1 + (1/2)(h/R)}{1 - (1/2)(h/R)}\right]^{-1} \tag{29}$$

In the present case, at the horizontal cross section ($\theta = 0$), the bending moment is given by

$$M = P(R - e) - M_1 = [1 - (2/\pi)]P(R - e) \tag{30}$$

The total normal stress at $\theta = 0$ is given by

$$\sigma = - (P/h) + \sigma_b \tag{31}$$

In the numerical example under consideration, equation (31) becomes

$$\sigma = - 437 + 18{,}835\ (0.0099356 + z)/(1.1875 + 2) \tag{32}$$

The numerical values are presented in the next to the last column in Table 2.

There is not a great deal of difference among all four stress distributions listed in Table 2. However, it is emphasized that the material is isotropic with $\nu = 0.35$, thus, the ratio of shear modulus to elastic modulus (G/E) is 0.3704. In contrast, for graphite/epoxy (T300/5208), this ratio is only 0.04368, according to the carefully correlated experimental results due to Knight (1982). Thus, it was decided to repeat the calculation of the present theory for a hypothetical ring constructed of hoop-wound graphite/epoxy and having the same geometry and loading as previously discussed. The resulting equation, instead of equation (26), is

$$\sigma_y = - 587 + 15,204\ z + 8,163\ z^2 + 119,000\ z^3 \qquad (33)$$

This calculation was based upon the following material properties reported by Knight (1982):

Hoop-direction elastic modulus	20.1×10^6 psi
Transverse shear modulus	0.378×10^6 psi

The resulting stress distributions are plotted in Fig. 4, where it is noted that the low shear modulus of the graphite/epoxy causes much higher maximum stresses than are present in the isotropic ring.

5. DISCUSSION

In the development of the equilibrium equations, (10)-(12), the integration method believed to have been originated by Yang (1965) was used. However, this approach leads to equilibrium equations which are only approximate for the present case, although they are exact for the cases of the classical thin and shear deformable theories. More accurate, yet much more complicated, equilibrium equations can be obtained by application of the principle of virtual work, such as used by Kraus (1967) for thin-shell theory. However, in the present case, this approach gives rise to five more generalized forces than the theory presented here. They are of the form

$$\int_{-h/2}^{h/2} (z^2\sigma_y,\ z^3\sigma_y,\ z\tau_{yz},\ z^2\tau_{yz},\ z^3\tau_{yz})\, dz$$

These are analogous to the higher-order generalized forces obtained by Tiffen and Lowe (1963) and Lo et al. (1977a,b) in their higher-order plate theories. See also the treatise by Librescu (1975). The present approximation is more in keeping with Levinson's plate and beam theories (1980, 1981). Thus, it may be more appropriate to classify the present theory as a more accurate simplified theory rather than as a true higher-order theory. It should be mentioned that Langhaar (1965) pointed out that higher-approximation theories are not necessarily more accurate. He cited the case of a curved cantilever beam for which Love's first-approximation theory (same as Winkler-Bach curved-beam theory) yielded the statically correct force quantities while his second-approximation theory gave results which violated static equilibrium grossly.

6. CONCLUSIONS

The new ring theory presented here has the following advantages:

1. In contrast to other shear-deformable ring theories it does not require separate *ad hoc* determination of a shear correction factor.

2. In contrast to other first-approximation ring theories, it allows for a nonlinear distribution of circumferential displacement through the thickness.

3. Relative to two- or three-dimensional elasticity theory and to higher-order ring theory, it is much simpler and much more amenable to engineering calculations.

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APPENDIX

The exact linear strain-displacement relations in shell coordinates are

$$\epsilon_{\theta} = (W + V_{,\theta})/(R + z) \quad ; \quad \epsilon_z = W_{,z}$$
$$\gamma_{\theta z} = V_{,z} + (W_{,\theta} - V)/(R + z) \qquad \text{(A-1)}$$

Using the displacement field of equations (1) in equations (A-1), one obtains

$$\epsilon = (R + z)^{-1}(w + v_{,} + z_{,} + z^{2}_{,} + z^{3}_{,})$$
$$\epsilon_z = 0 \quad ; \quad \gamma_z = (R + z)^{-1}(w_{,} - v - z - z - z) + + 2z + 3z \qquad \text{(A-2)}$$

Enforcing $\gamma_z(, \pm \frac{1}{2} h) = 0$ thus requires that the following four equations be satisfied:

$$w_{,} - v - (h^2/4) = 0$$
$$+ (h^2/4) = 0 \quad ; \quad = 0 \quad ; \quad + (3h^2/4) = 0 \qquad \text{(A-3)}$$

It is obvious that all four of these cannot be satisfied except in the trivial case, $= = = 0$ and $w_{,} = v$.

Table 1. Conditions for experiments of Srinath and Acharya (19540

Dimensional Data		Loading and Material Data
Ring depth	h = 0.375 in.	Loading P = 164 lb/in.
Ring mean radius	R = 1.1875 in.	
Ratio	h/R = 0.3158	Poisson's ratio ν = 0.35*

*Estimated

Table 2. Numerical results for distribution of circumferential normal stress (psi)

	Present Theory			
z/h	$S \to \infty$	Actual S	Winkler-Bach Theory	Measured
-0.500	-3,246	-3,371	-3,781	-3,550
-0.233	-1,871	-1,882	-1,765	-1,720
0	-517	-525	-279	Not reported
0.042	-264	-273	-37	-70
0.291	1,301	1,322	1,292	1,300
0.500	2,656	2,809	2,268	2,350

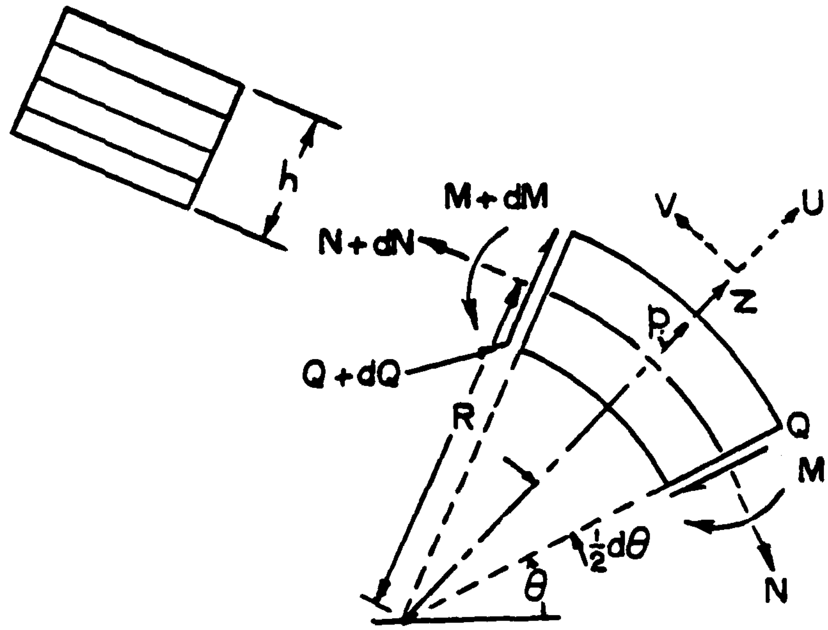

Fig.1 . Coordinates, displacements, forces and moments acting on a differential element of a ring.

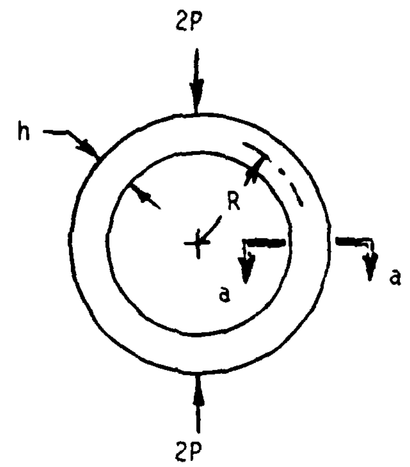

Fig. 2. A diametrically loaded thick circular ring.

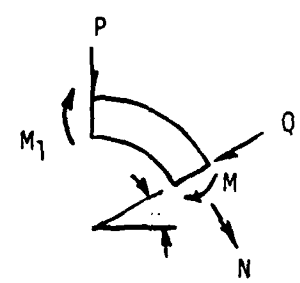

Fig. 3. Free-body diagram of ring segment.

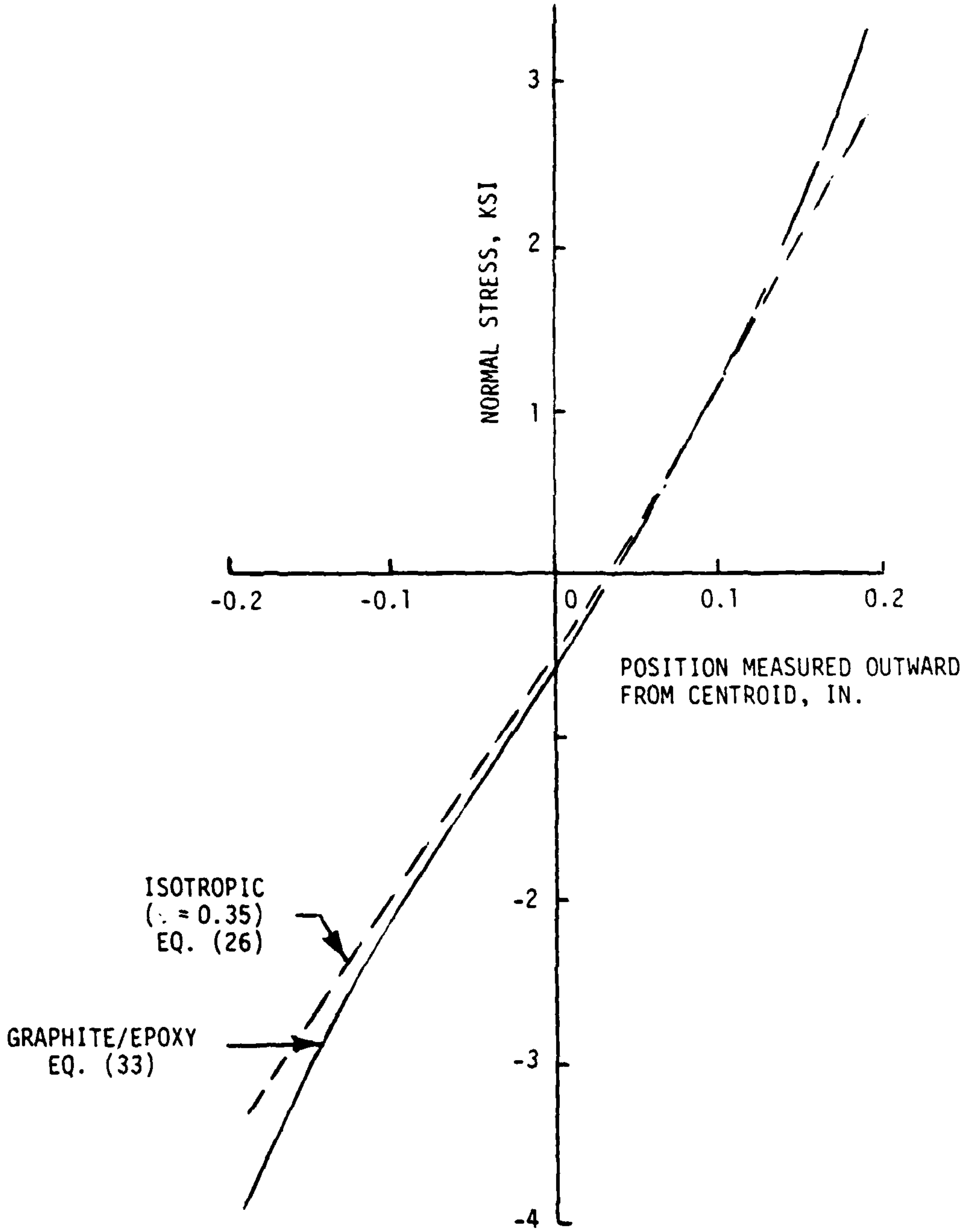

Fig. 4. Normal-stress distributions for isotropic and graphite/epoxy rings loaded in diametral compression.

PREVIOUS REPORTS ON THIS CONTRACT

Project Rept. No.	Issuing University Rept. No.*	Report Title	Author(s)
1	OU 79-7	Mathematical Modeling and Micromechanics of Fiber Reinforced Bimodulus Composite Material	C.W. Bert
2	OU 79-8	Analyses of Plates Constructed of Fiber-Reinforced Bimodulus Materials	J.N. Reddy & C.W. Bert
3	OU 79-9	Finite-Element Analyses of Laminated Composite-Material Plates	J.N. Reddy
4A	OU 79-10A	Analyses of Laminated Bimodulus Composite-Material Plates	C.W. Bert
5	OU 79-11	Recent Research in Composite and Sandwich Plate Dynamics	C.W. Bert
6	OU 79-14	A Penalty Plate-Bending Element for the Analysis of Laminated Anisotropic Composite Plates	J.N. Reddy
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9	OU 79-20	Effects of Shear Deformation and Anisotropy on the Thermal Bending of Layered Composite Plates	J.N. Reddy & Y.S. Hsu
10	OU 80-1	Analyses of Cross-Ply Rectangular Plates of Bimodulus Composite Material	V.S. Reddy & C.W. Bert
11	OU 80-2	Analysis of Thick Rectangular Plates Laminated of Bimodulus Composite Materials	C.W. Bert, J.N. Reddy, V.S. Reddy, & W.C. Chao
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27	OU 82-3	Research on Dynamics of Composite and Sandwich Plates, 1979-81	C.W. Bert
28	OU 82-4 & VPI 82.20	Mechanics of Bimodular Composite Structures	C.W. Bert & J.N. Reddy

*OU denotes the University of Oklahoma; VPI denotes Virginia Polytechnic Institute and State University.

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Project Rept. No.	Issuing University Rept. No.	Report Title	Author(s)
29	VPI 82 19	Three-Dimensional Finite Element Analysis of Layered Composite Structures	W.C. Chao, N.S. Putcha, J.N. Reddy
30	OU 82-5	Analyses of Beams Constructed of Nonlinear Materials Having Different Behavior in Tension and Compression	C.W. Bert & F. Gordaninejad
31	VPI 82.31	Analysis of Layered Composite Plates by Three-Dimensional Elasticity Theory	J.N. Reddy & T. Kuppusamy
32	OU 83-1	Transverse Shear Effects in Bimodular Composite Laminates	C.W. Bert & F. Gordaninejad
33	OU 83-2	Forced Vibration of Timoshenko Beams Made of Multi-modular Materials	F. Gordaninejad & C.W. Bert
34	VPI 83.34	Three-Dimensional Analysis of Composite Plates with Material Nonlinearity	T. Kuppusamy, A. Nanda, & J.N. Reddy
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18. SUPPLEMENTARY NOTES

A preliminary version of this paper was presented as an invited paper at the 17th Israel Conference on Mechanical Engineering, Tel Aviv University, Tel Aviv, Israel, July 12-14, 1983.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Composite materials, moderately thick rings, rings (structural), shear deformable rings, static loading, transverse shear deformation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A new theory for the in-plane elastic behavior of shear-deformable ring-type structures, i.e., curved beams and complete rings, is presented. This theory falls between Bresse-Timoshenko-type ring theory, in which the shear deformation is assumed to be distributed uniformly through the thickness and corrected by a shear correction factor determined in an *ad hoc* fashion, and two- or three-dimensional elasticity theory. As an example, the theory is applied to the problem of a diametrically loaded thick ring. The predicted

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20. Abstract - Cont'd

normal-stress distribution is in excellent agreement with published results in the literature obtained by photoelastic measurements.

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